Announcements

1) Nissan recruiting,
Chesebrough Auditorium
in Ann Arbor on campus,
talk 6-7 by CEO w/
reception following

2) Organizing study groups -
can use Piazza to find
compatible times.

3) Official Office hours: MW 9-9:30
M 1-2 T 2:30-3:30

Example 1: (chemistry)

Book # 8, Section 1.6

Limestone ($CaCO_3$) neutralizes the hydronium ion (H_3O) in acid rain by the following equation:

$$H_3O + CaCO_3 \rightarrow H_2O + Ca + CO_2$$

Balance the equation.

This means: find whole numbers (integers) x_1, x_2, x_3, x_4, and x_5 with

$$\boxed{x_1\, H_3O + x_2\, CaCO_3 \rightarrow x_3\, H_2O + x_4\, Ca + x_5\, CO_2}$$

i.e. the number of atoms of each element must balance out.

Make a matrix.

1st row = number of Hydrogen atoms

2nd row = number of Carbon atoms

3rd row = number of Calcium atoms

4th row = number of Oxygen atoms

$$x_1 H_3O = x_1 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_2 CaCO_3 = x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

$$x_3 H_2O = x_3 \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_4 Ca = x_4 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$x_5 CO_2 = x_5 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

Equation becomes

$$x_1 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

$$+ x_3 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ 0 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrix form

$$\begin{bmatrix} 3 & 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 3 & -1 & 0 & -2 & 0 \end{bmatrix}$$

rref gives $x_1 - 2x_5 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

So $x_1 = 2x_5$

$x_2 = x_5$

$x_3 = 3x_5$

$x_4 = x_5$

$(x_5 = x_5)$

Make a (nonzero) choice of whole number for x_5:

If $x_5 = 1$, $x_2 = x_4 = 1$

and $x_1 = 2$, $x_3 = 3$.

General solution:

$$\begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

where s is any whole number including zero.

Notation: ($\mathbb{R}^n$)

$\mathbb{R}^n$ for n a nonzero whole number denotes all n-vectors: anything of the form $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

where v_i is a real number, $1 \leq i \leq n$, is in $\mathbb{R}^n$.

Recall: We can add and scalar-multiply n-vectors.

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix},$$

then $$v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{bmatrix}.$$

If c is any real number

$$c \cdot v = \begin{bmatrix} c v_1 \\ c v_2 \\ \vdots \\ c v_n \end{bmatrix}$$

Example 2:

$\begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -11 \\ 22 \end{bmatrix}$ are in $\mathbb{R}^2$,

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} -11 \\ 22 \end{bmatrix} = \begin{bmatrix} -9 \\ 27 \end{bmatrix}$$

and

$$16 \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 32 \\ 80 \end{bmatrix}$$

Why either one solution or infinitely many?

If x, y are in $\mathbb{R}^n$, c is in $\mathbb{R}$, A is any $m \times n$ matrix,

1) $A(x+y) = Ax + Ay$

2) $A(cx) = c\,Ax$

If $Ax = b$ and $Ay = b$,

let c be any number in $(0,1)$. If $x \neq y$,

$$A(cx + (1-c)y)$$

$$= A(cx) + A((1-c)y) \quad \text{(rule 1)}$$

$$= c\,Ax + (1-c)\,Ay \quad \text{(rule 2)}$$

$$= c\,b + (1-c)\,b \quad \text{(given)}$$

$$= (c + (1-c))\,b = b$$

This is the level of proof you will have to deal with in this class.