

Announcements

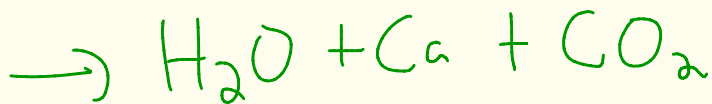
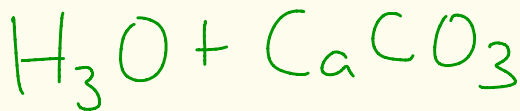
- 1) Nissan recruiting,
Chesebrough Auditorium
in Ann Arbor on campus,
talk 6-7 by CEO w/
reception following
- 2) Organizing study groups -
can use Piazza to find
compatible times.
- 3) Official Office hours: MW 9-9:30
M 1-2 T 2:30-3:30

Example 1 : (Chemistry)

Book # 8, Section 1.6

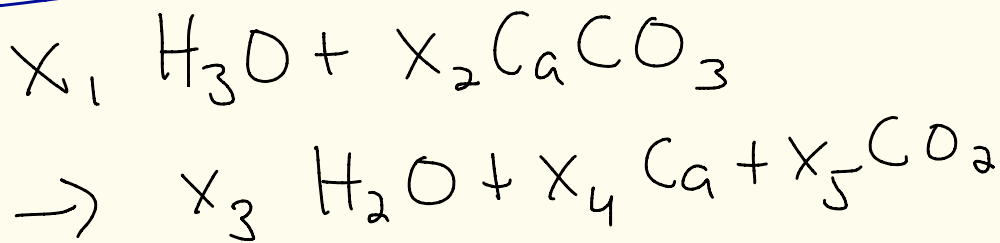
Limestone (CaCO_3)

neutralizes the hydronium ion (H_3O^+) in acid rain by the following equation:



Balance the equation.

This means: find
whole numbers (integers)
 x_1 , x_2 , x_3 , x_4 , and x_5 with



i.e. the number of atoms
of each element must
balance out.

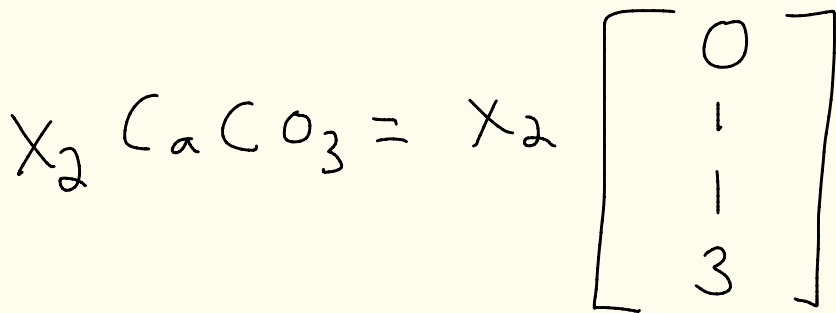
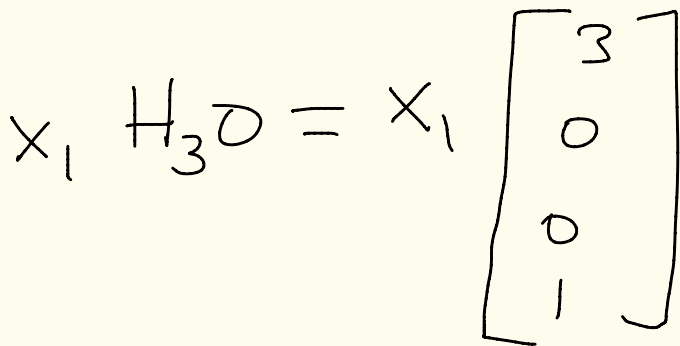
Make a matrix.

1st row = number of Hydrogen atoms

2nd row = number of Carbon atoms

3rd row = number of Calcium atoms

4th row = number of Oxygen atoms



$$X_3 \text{H}_2\text{O} = x_3 \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$X_4 \text{Ca} = x_4 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$X_5 \text{CO}_2 = x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 2 \end{bmatrix}$$

Equation becomes

$$X_1 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} + X_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

$$+ X_3 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} + X_4 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} + X_5 \begin{bmatrix} 0 \\ -1 \\ 0 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrix form

$$\begin{bmatrix} 3 & 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 3 & -1 & 0 & -2 & 0 \end{bmatrix}$$

rref gives $x_1 - 2x_5 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$\text{So } x_1 = 2x_5$$

$$x_2 = x_5$$

$$x_3 = 3x_5$$

$$x_4 = x_5$$

$$(x_5 = x_5)$$

Make a (nonzero) choice
of whole number for x_5 :

$$\text{If } x_5 = 1, \quad x_2 = x_4 = 1$$

$$\text{and } x_1 = 2, \quad x_3 = 3.$$

General solution:

$$\begin{bmatrix} 2 \\ 1 \\ 3 \\ \vdots \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 3 \\ \vdots \end{bmatrix}$$

Where s is any
whole number including
zero.

Notation: (\mathbb{R}^n)

\mathbb{R}^n for n a nonzero
whole number denotes all

n -vectors: anything

of the form
$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

where v_i is a real number, $1 \leq i \leq n$,
is in \mathbb{R}^n .

Recall: We can add and scalar-multiply n -vectors.

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix},$$

then $v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{bmatrix}$.

If c is any real number

$$c \cdot v = \begin{bmatrix} c v_1 \\ c v_2 \\ \vdots \\ c v_n \end{bmatrix}$$

Example 2:

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -11 \\ 22 \end{bmatrix} \text{ are in}$$

$$\mathbb{R}^2,$$

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} -11 \\ 22 \end{bmatrix} = \begin{bmatrix} -9 \\ 27 \end{bmatrix}$$

and

$$16 \cdot \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 32 \\ 80 \end{bmatrix}$$

Why either one solution or infinitely many?

If x, y are in \mathbb{R}^n , c is in \mathbb{R} ,
 A is any $m \times n$ matrix,

$$1) A(x+y) = Ax + Ay$$

$$2) A(cx) = cAx$$

If $Ax = b$ and $Ay = b$,

let c be any number
in $(0, 1)$. If $x \neq y$,

$$A(cx + (1-c)y)$$

$$= A(cx) + A((1-c)y) \text{ (rule 1)}$$

$$= cAx + (1-c)Ay \text{ (rule 2)}$$

$$= cb + (1-c)b \text{ (given)}$$

$$= (c + (1-c))b = b$$

This is the level of proof
you will have to deal with
in this class.